Influence Propagation: Patterns, Model and a Case Study

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Abstract. When a free, catchy application shows up, how quickly will people notify their friends about it? Will the enthusiasm drop exponentially with time, or oscillate? What other patterns emerge?
Here we answer these questions using data from the Polly telephone-based application, a large influence network of 72,000 people, with about 173,000 interactions, spanning *500MB* of log data and *200 GB* of audio data.
We report surprising patterns, the most striking of which are: (a) the FIZZLE pattern, i.e., excitement about Polly shows a power-law decay over time with exponent of -1.2; (b) the RENDEZVOUS pattern, that obeys a power law (we explain RENDEZVOUS in the text); (c) the DISPERSION pattern, we find that the more a person uses Polly, the fewer friends he will use it with, but in a reciprocal fashion. Finally, we also propose a generator of influence networks, which generate networks that mimic our discovered patterns.

Keywords: social network mining, influence network, influence patterns.

1 Introduction

How will a catchy phone application propagate among people? Will the excitement about it spike, oscillate, or decay with time?

Information cascades, like the above one, appear in numerous settings, like blogs, trending topics in social networks, memes, to name a few. Social influence has been a topic of interest in the research community [38,30,16,23,21,9,4,8,11,27] because of the rise of various on-line social media and social networks. In this work, by *social influence* we refer to the fact that "individuals adopt a new action because of others".

Our current work tries to answer all these questions in a large dataset of hundreds of thousands of interactions. We obtained access to Polly data. Polly is a voice, telephone-based application which allows the sender to record a short message, choose among six funny manipulations of distorting his voice-message (faster, slower, high-pitch, etc.), and forward the modified recording to any of his friends[1]. Polly was devised as a platform for disseminating and popularizing voice-based information services for low-skilled, low-literate people in the developing world. We focus in two main problems, described informally as follows:

[1] For a brief video introduction to Polly, including demos of different voice effects, see http://www.cs.cmu.edu/ Polly/

Informal Problem 1 (Pattern Discovery) *Consider a real-world influence network:* **Given** *who influences whom, and when,* **find** *general influence patterns this network obeys.*

Informal Problem 2 (Generator) *Create a realistic influence-network-generator:*
– **Given** *a friendship social network (who-likes-whom)*
– **Design** *a simple, local propagation mechanism*
– *so that we can generate realistic-looking influence networks.*

By "realistic" we mean that the resulting influence networks match our discovered patterns.

Figure 1 gives examples of a social network (who is friends with whom, in gray, directed edges), and a possible influence network (who sends messages to whom - in red; directed, time-stamped, multi-edges). For simplicity, only edges between 1 and 2 are shown with time-stamp and multi-edge structure.

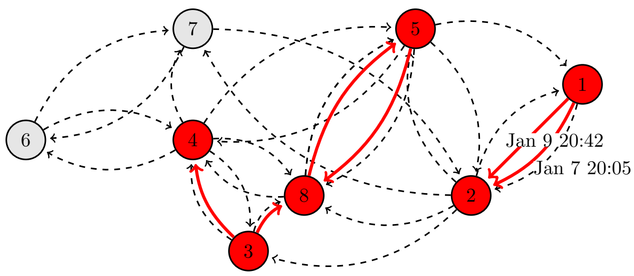


Fig. 1: Illustration of a 'base network' (in gray) and a possible 'influence network' (= cascades; in red). Here, the initial nodes ('seeds') are nodes 1 and 3.

The contributions of this work are the following:

– **Discovery** of three new patterns (laws): the FIZZLE, RENDEZVOUS, and DISPERSION pattern (Sections 3, 4, 5, respectively);
– **Generator and Analysis:** We propose a local, efficient propagation mechanism that simulates an influence graph on top of existing social network datasets (Enron, Facebook) or synthetic social network datasets [12]. Figure 1 illustrates the process of simulating influence network. We also did analysis on the DISPERSION pattern.

The importance of the former contribution is that patterns can help marketers and sociologists understand how influence propagates in a social network; they can also help spot anomalies, like spammers, or faulty equipment.

The importance of our second contribution is that a realistic generator is valuable for what-if scenarios, and reproducibility: publicly-available influence network datasets are notoriously difficult to obtain, due to privacy and corporate regulations; a good generator can serve as proxy.

Reproducibility

For privacy reasons, the Polly dataset that we used in this paper is not public. Thus, for reproducibility, we present experiments on public data (such the Enron Email network [22,1], and Facebook [2]) which exhibit similar behavior like our dataset. We also make our code open-source at: `https://github.com/yibinlin/inflood_generator/`.

Next, we describe the dataset (Sec. 2), our discoveries (Sec. 3, 4 and 5), our generator (Sec. 6), the related work (Sec. 7) and conclusions (Sec. 8).

2 Dataset Used

The dataset comes from Polly [34,33,32], a simple, telephone-based, voice message manipulation and forwarding system that allows a user to make a short recording of his voice, optionally modify it using a choice of funny sound effects, and have the modified recording delivered to one or more friends by their phone numbers. Each friend in turn can choose to re-forward the same message to others, and/or to create a new recording of his own. Using entertainment as motivation, Polly has been spreading virally among low-literate users in South Asia, training them in the use of speech interfaces, and introducing them to other speech-based information services.

The dataset comes from the first large-scale deployment of Polly in Lahore, Pakistan. After being seeded with only 5 users in May, 2012, Polly spread to 72,341 users by January, 2013. There are 173,710 recorded interactions , spanning *500 MB* of real-world message delivery log and *200 GB* of audio data. However, this analysis focuses only on the forwarding of user recorded messages. In what follows, we denote a user with a node and a forwarded message with a directed and dated edge. Hence we view our dataset as an influence network.

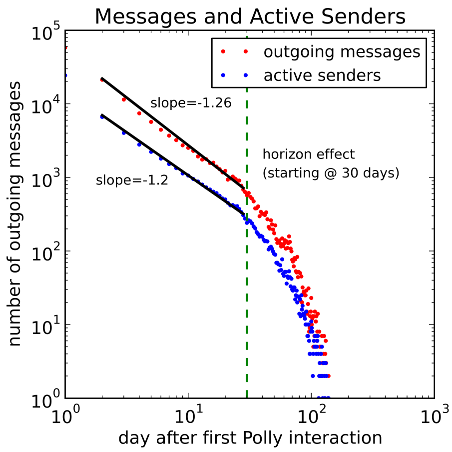
We have IRB[2]-approved access to the full, though anonymized, logs of interaction.

3 Discovered Pattern (P1): FIZZLE

Users may have been introduced to Polly by receiving a forwarded message from one of their friends, or simply by “word of mouth”. Many such users may in turn call the system, experiment with it, and possibly send messages to their own friends. Most such users cease interacting with the system within a few days. Still, a significant number of users stay with the system for a long time. How does these users’ activity change over time?

In the following analysis, we define a user’s “system age” as the number of days elapsed after the user successfully sends out first message. Moreover, the *active senders* after n days are defined as the users who actively send out messages on the n-th day after they sent their first message. Figure 2 depicts the FIZZLE pattern: the number of active senders (that is, users that still send messages to their friends) vs. their system age. It also shows the count of messages they sent, as a function of their system age.

[2] Institutional Review Board



Activity (number of messages (red), remaining users (blue)) vs system age.

Fig. 2: The FIZZLE pattern (P1) - best viewed in color: the number of messages sent (in red) and count of active senders (in blue), versus system age. In both cases, the excitement follows a power law with exponent ≈ -1.2. The horizon effect is explained in Section 3.

Both follow power-law distribution with exponents of -1.2 and -1.26, respectively. This observation agrees with earlier results of the behavior of elapsed time in communication patterns (see [30]): there, Oliveira et al reported shows similar power-law patterns in mail and e-mail correspondences, but with slightly different exponents (1.5 and 1).

Observation 1 (P1) *The number of active senders $c(t)$ at system age t follows*

$$c(t) \propto t^{\alpha} \tag{1}$$

where $\alpha \approx -1.2$. Similarly for the count of messages $m(t)$ at system age t.

Horizon effect: In order to get accurate information about the FIZZLE pattern, new users who are introduced to the system later than 110th day after it was launched were excluded. In this paper, messages delivered within the first 140 days are analyzed. In other words, all the users shown in Figures 2 have passed "system age" of 30 (no matter whether they are still active or not) because they were introduced to the system at least 30 days before the end of our analysis scope. This is exactly the reason for the deviation from power-law "system ages" of 30 and above are unfairly handicapped.

The detailed power-law linear regression results for the FIZZLE pattern, as well as all our upcoming patterns, are listed in Table 1. Notice that they all have extremely high correlation coefficient (absolute value ≥ 0.95).

4 Discovered Pattern (P2): RENDEZVOUS

In a directed network, propagation from one source can take multiple paths to the same destination node. Of particular interest to us are two paths that diverge for a while (with

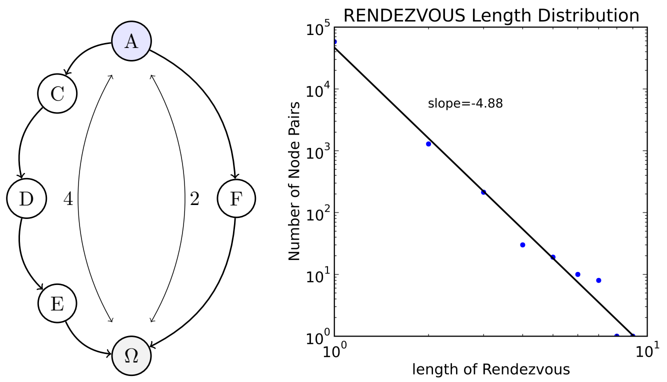
Table 1: Summary of Power Laws Observed in Our Dataset

Pattern	Slope k	Correlation Coefficient r
P1 The FIZZLE pattern (number of remaining users) k_1	-1.2	-0.994
P1 The FIZZLE pattern (number of phone calls) k_3	-1.26	-0.996
P2 The RENDEZVOUS pattern k_2	-4.88	-0.992

no intermediate connections between them) before they re-converge – an event which we here call RENDEZVOUS. This event type corresponds to diffusion into different social circles (e.g. a-friend-of-a-friend... of-my-friend, whom I am unlikely to know), followed by convergence. The prevalence of such re-convergences can shed light on the effective population size. In a large country like Pakistan (180 million people), the effective population size for our system may vary widely and is unknown a priori. Thus, we are interested in the prevalence of RENDEZVOUS as a function of the shortest path to the most recent common ancestor of two parents of a node, where path length and recency are both measured in terms of number of edges, rather than time. A node with k parents gives rise to $k \cdot (k-1)/2$ different RENDEZVOUS. Taking into account the edge from the common child to its parents, we have the following definition:

Definition 1 (n-RENDEZVOUS:) *An n-RENDEZVOUS is defined as a RENDEZVOUS where the* shorter *path from the two parents to their most recent ancestor is of length $n-1$.*

For example, Fig. 3(a) shows a RENDEZVOUS of length 2: the shortest leg from the final node Ω to the starting node A, is n=2 hops long.



(a) A 2-RENDEZVOUS. (b) Distribution of RENDEZVOUS lengths

Fig. 3: (a) Example of a 2-RENDEZVOUS. (b) Distribution of RENDEZVOUS lengths follows a power law with exponent -4.88.

Figure 3(b) shows that the length distribution of RENDEZVOUS' in our dataset follows a power-law . Most of RENDEZVOUS have a length of 1, meaning that one of the parents *is* the most recent common ancestor, because it has a direct link to the other parent.

Observation 2 (P2) *The number of* RENDEZVOUS' $n(l)$ *at* RENDEZVOUS *length* l *follows*

$$n(l) \propto l^{\beta} \tag{2}$$

where $\beta \approx -4.88$.

5 Discovered Pattern (P3): DISPERSION

Let the "reciprocal activity" between two users be the smaller of the number of messages sent between them in either direction. Let the "activity profile" of a user be $\{m_1, m_2, m_3, ..., m_F\}$ $(m_1 > m_2 > m_3 > ... > m_F > 0)$, where m_i is the reciprocal activity between a user and one of his recipients.

Definition 2 (DISPERSION) *The* DISPERSION D *of a user with activity profile* $\{m_1, m_2, m_3, ..., m_F\}$ *is defined as the entropy* H *of the normalized count distribution:*

$$D(m_1, m_2, ..., m_F) = -\sum_{r=1}^{F} P_r * ln(P_r)$$

Where $P_r = {}^{m_r}/\sum_{k=1}^{F} m_k$.

Therefore, if a user has a high DISPERSION, she sends messages her friends more evenly than other users with the same number of friends, but lower DISPERSION.

Figure 4(a) shows that the real DISPERSION (entropy) is smaller than the "maximum dispersion" where a user sends messages each of her friends evenly. This means that long-term Polly users on average exhibit the DISPERSION pattern when they send messages to their friends.

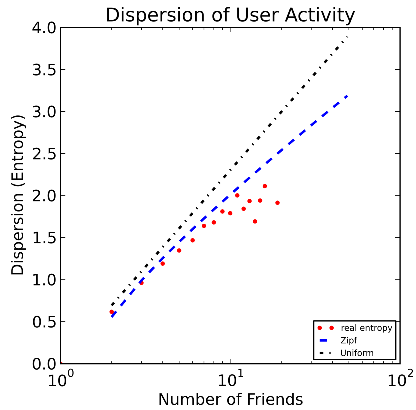
We can explain the DISPERSION behavior using a closed-form formula, under the assumption that people send messages to their friends following a Zipf's distribution, which implies $P_r \propto 1/r$, to be specific, $P_r \approx 1/(r \times ln(1.78F))$[35, p. 33]. Based on this, we can derive that if we use integral as an approximation of the sum part of the entropy calculation:

Lemma 1. *The entropy* H *of a Zipf's distribution is given by:*

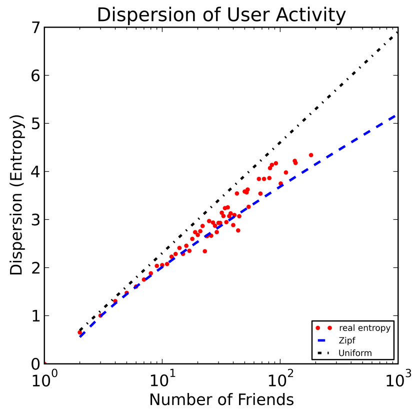
$$H \approx (C \times ln^2(F) + K \times ln(F) lnln(1.78F)), \quad \textit{where } F > 1. \tag{3}$$

The proof is omitted for brevity.

Observation 3 (P3) *Dispersion pattern can be modelled well by Zipf's law in our dataset.*



(a) DISPERSION pattern in our dataset



(b) DISPERSION pattern in Experiment E_1 (Enron)

Fig. 4: (a) DISPERSION pattern found in the influence network. (b) DISPERSION pattern found in simulated influence network in Experiment E_1, see Section 6.

The mathematical analysis shows that the "friend contact" distribution of a user with F reciprocal friends will follow an expected entropy value proportional to the square of logarithm of F $(ln^2(F))$ other than $(ln(F))$ when we assume the distribution follows uniform distribution. The predicted entropy of Eq (3) matches reality much better than the uniformity assumption. As shown in Figure 4(a), the predicted entropy (the dashed blue curve) is a better match for the real data (red dots), while the uniformity assumption leads to the black-dotted line.

6 INFLOOD GENERATOR: Algorithm and Evaluation

First, we formally define the *base* and *influence* network:

Definition 3 (Base Network) *A Base Network (V_{base}, E_{base}) is the underlying social network of all people who are related to social information cascades. V_{base} is a set of individuals. E_{base} is a set of directed, weighted edges. The weights represent the strength of connections.*

Definition 4 (Influence Network) *An Influence Network (V_{infl}, E_{infl}) shows which node sent a system message to which node, and when. V_{infl} is a set of individuals. E_{infl} is a set of directed, timestamped edges of which the weight shows the number of times a node has been notified of the influence by another node.*

In our model, $V_{infl} \subseteq V_{base}$, $E_{infl} \subseteq E_{base}$, i.e., individuals can only be influenced by others they know.

We model all patterns by using INFLOOD GENERATOR. As mentioned above, Polly can be viewed as an influence network where people are notified of it from their base-network friends. After the notification, people may start forwarding messages.

Why we need a generator The best way to verify all our three patterns (FIZZLE, RENDEZVOUS and DISPERSION) is to study **other** influence network datasets. However, they are difficult to obtain, and some of them lack time stamp information (See Sec 1).

Details of INFLOOD GENERATOR The pseudo code of our influence-network generator, is given in Algorithm 1. In more details, on day 0, s_0 seed nodes ($s_0 = 5$) in the social network $G1$ (e.g., Facebook or Enron) are notified of Polly. Then, each following day t, every person u who has been notified of Polly has a probability, $P_{u,t}$, of calling some of her friends via Polly. The friends are sampled from the outgoing edges from $G1$ *independent of* no matter whether they have been already notified.

Input: $G1 =$ A base network, $T =$ simulated days, $\alpha =$ power-law decay factor, $P_{u,0}$: first-day infection probability of user u, $day = 0$.
Output: An influence network $G2$ on top of $G1$
begin
    $notifiedUsers \longleftarrow$(5 randomly chosen people in $G1$ with notified day $t_u = 0$);
    $day \longleftarrow 0$;
    while $day < T$ **do**
        for *each user* u *in* $notifiedUsers$ **do**
            $coin \longleftarrow TAIL$;
            if $random() < P_{u,t}$ *in Equation 4* **then**
                $coin \longleftarrow HEAD$;
            while $coin == HEAD$ **do**
                f = a friend sampled from $G1$;
                if f *is not in* $notifiedUsers$ **then**
                    add f into $notifiedUsers$ with $t_u = day$
                else
                    //User u has sent a message to user f, Record this interaction.
                    //Details are omitted for simplicity.
                if $random() >= P_{u,t}$ **then**
                    $coin \longleftarrow TAIL$;
        $day \longleftarrow day + 1$;

Algorithm 1: Pseudo Code of INFLOOD GENERATOR

The probability of a user u telling other people about Polly is given by

$$P_{u,t} = P_{u,0} \times (day - t_u)^{-1.0\times\alpha}, day > t_u \tag{4}$$

, where α parameter affects the exponent of the power-law that governs the decay of infection activities, and $P_{u,0}$ is defined the *first-day infection probability*.

Estimating $P_{u,0}$ $P_{u,0}$ depends on the total weight (number of communication messages) a user (node) has in base network $G1$. In fact, $P_{u,0}$ is set so that, in expectation, the number of edges user u makes on the first day $msg_{u,0}$ is proportional to the user's total weight of out-going connection strength in base network $G1$, i.e.

$$msg_{u,0} = c \times \sum_{v \in V_{G1}} w_{u,v} \tag{5}$$

Note that in Algorithm 1, the process of sending messages is a geometric distribution. Hence we can get $msg_{u,0} = 1/(1-P_{u,0})$ in expectation.

Hence, we set the $P_{u,0}$ to be:

$$P_{u,0} = 1 - \frac{1}{c \times \sum_{v \in V_{G1}} w_{u,v}} \tag{6}$$

This ensures that a high weight node has more simulated edges. It is also more realistic, because more social people may spread messages more easily. In our setting, $c = \frac{1}{4}$, i.e. a user will contact a quarter of her $G1$ out-degrees in expectation on the first day. This formula is based on experimental observations.

Evaluation of INFLOOD GENERATOR We tested INFLOOD GENERATOR in a number of networks. Here we use communication networks, such as Facebook, to be approximations of "real" base network. The results are presented in Table 2.

Table 2: Results of INFLOOD Simulations

Experiment	**Base Network** $G1$	$G2\ \lvert V \rvert$	$G2\ \lvert E \rvert$	FIZZLE slope k_1	RENDEZVOUS slope k_2
Polly	N/A	$72,341$	$173,710$	-1.2	-4.88
E_1	Enron [22,1]	$19,829$	$227,659$	-1.16	-8.39
E_2	Slashdot [3,17]	$6,880$	$19,781$	-1.18	-6.11
E_3	Facebook [2]	$22,029$	$222,686$	-1.16	-7.65

In all experiments, $\alpha = 1.17$, and the number of simulated days is $T = 140$.

In all cases, the correlation coefficients $|r|$ were high ($|r| > 0.93$). The FIZZLE slopes k_1 are calculated based only on the first 30 days of interactions of each user, exactly as we did for the real, Polly dataset. Recall that k_1 is the slope of the FIZZLE pattern, that is, the slope of the number of remaining active users, over time, in log-log scales. For the RENDEZVOUS pattern, the k_2 slope varies between experiments. This may be due to the small count of data points, see Fig 3(b). We also tested the INFLOOD GENERATOR on synthetic datasets, such as Erdös-Rényi graphs of various parameter settings. Notice that the RENDEZVOUS pattern is violated: the Erdös-Rényi graphs do *not* follow a power-law in their RENDEZVOUS plots.

Because the INFLOOD GENERATOR graph is big, we observe the DISPERSION pattern in Experiment E_1. Figure 4(b) shows that the entropy footprint grows well with the Zipf's distribution curve for users who have less than 50 friends. When the number of friends goes beyond 50, the entropy footprints seem less regular as the number of samples decreases.

Again, the INFLOOD GENERATOR code is open source, see Sec 1.

7 Related Work

Static graph patterns. These include the legendary 'six-degrees of separation' [29]; the skewed degree distribution [13], specially for telephone graphs [5]; the power law tails in connected components distributions; the power law PageRank distributions and bimodal radius plots [20]; the super-linearity rules [28], triangle patterns [39,19]. This list is by no means exhaustive; see [10] for more patterns. Algorithms for detecting these patterns have been proposed by multiple research teams, such as [18].

Temporal and influence patterns. Work on this topic encompasses the shrinking diameter and densification [25]; the power law for the mail response times of Einstein and Darwin, [30]; analysis of blog dynamics [16,26], and discovery of core-periphery patterns in blogs and news articles [15]; viral marketing [23,21]; meme tracking [24]; reciprocity analysis [14,6]; analysis of the role of weak and strong ties in information diffusion in mobile networks [31]; identification of important influencers [36]; prediction of service adoption in mobile communication networks [37]; information or cascade diffusion in social networks [9,4,8,38]; linguistic change in online forums, and predicting the user's lifespan based on her linguistic patterns [11]; peer and authority pressure in information propagation [7].

However, none of the above works reports anything similar to our discoveries, the RENDEZVOUS and the DISPERSION patterns.

8 Conclusions

We study a large, real influence network induced by the Polly system, with over 70,000 users (nodes), 170,000 interactions (edges), distilled from 500MB of log data and 200GB of audio data. Polly is a free, telephone-based, voice message application that has been deployed and used in the real world. Our contributions are as follows:

1. **Discovery** of new patterns in Polly:
   - P1: the 'enthusiasm' drops as a power law with time.
   - P2: The RENDEZVOUS pattern shows a power-law distribution.
   - P3: The DISPERSION pattern of users behaves like a Zipf distribution;
2. **Generator and Analysis**:
   - We propose the INFLOOD GENERATOR algorithm, which matches the observed patterns (P1, P2 and P3) in various communication networks. The code is open-sourced at `https://github.com/yibinlin/inflood_generator/`.
   - We give the derivation for the observed DISPERSION pattern

With respect to future work, a fascinating research direction is to estimate the underlying population size of our dataset, from the statistics of the RENDEZVOUS pattern.

9 Acknowledgements

The authors would like to thank Naoki Orii in CMU for suggestions and proof-reading.

This material is based upon work supported by the National Science Foundation under Grant No. IIS-1217559. Funding was also provided by the U.S. Army Research Office (ARO) and Defense Advanced Research Projects Agency (DARPA) under Contract Numbers W911NF-11-C-0088 and W911NF-09-2-0053. The content of the information in this document does not necessarily reflect the position or the policy of the Government, and no official endorsement should be inferred. The U.S. Government is authorized to reproduce and distribute reprints for Government purposes notwithstanding any copyright notation here on.

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